MOMENT DISTRIBUTION METHOD

by Prof. Hardy Cross.

Carry Over Theorem: When a moment M is applied to produce rotation without translation at the near supported end B of a beam whose farther end 'A' is fixed, the carry over moment M_a at the farther end is one-half the applied moment 'M' and is of same sense as that of the applied moment

Stiffness of member AB: is Moment to be applied at B, so as to produce unit slope at B equal to $\frac{4EI}{l}$.

Slope $i = \frac{Ml}{4EI}$ (for propped cantilever).

Moment $M = \frac{4EI}{l} i$.

When a structural member of uniform section is subjected to a moment at one end only; then the moment req^d so as to rotate that end to produce unit slope is called stiffness of the member.

For propped cantilever stiffness $= \frac{4EI}{l}$

S.S. Beam with couple M at B

$i_b = \frac{Ml}{3EI}$ or $M = \frac{3EI i_b}{l}$

Moment req^d to be applied at B to produce slope i_b is;

<u>Distribution theorem</u> Moment which is applied to a structural joint to produce rotation without translation gets distributed among the connecting members at the joint in the same proportion as their stiffnesses.

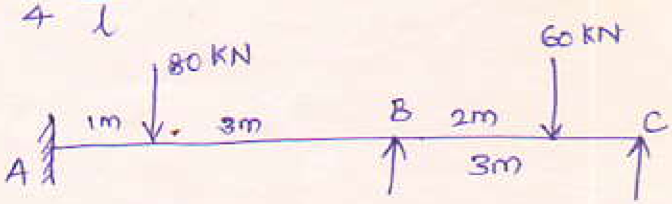
<u>Distribution factor</u> for a member at a joint is ratio of

$$\frac{\text{stiffness of the member}}{\text{Total stiffness of all members meeting at the joint}}$$

(*) Relative stiffness of a member at a joint whose farther end is fixed is I/l

(*) Relative stiffness of a member at a joint whose farther end is hinged or simply supported $= \frac{3}{4} \cdot \frac{I}{l}$

<u>Prob</u>: Find support moments and draw BMD.



$I_{ab} : I_{bc} = 2:1$

<u>Sol</u>: <u>Fixed end moments</u>

Consider each span as a seperated fixed beam.

$$\bar{M}_{ab} = -\frac{Wab^2}{l^2} = \frac{-80 \times 1 \times 3^2}{4^2} = -45 \text{ KN·m}$$

$$\bar{M}_{ba} = \frac{+Wba^2}{l^2} = +15 \text{ KN·m}$$

$$\bar{M}_{bc} = -\frac{Wab^2}{l^2} = -13.33 \text{ KN·m}$$

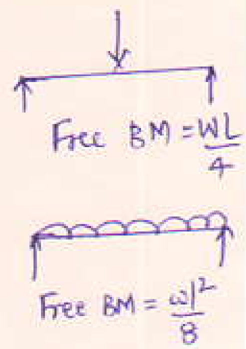
$$\bar{M}_{cb} = +26.67 \text{ KN·m}$$

Distribution Factors

Joint	Member	Relative stiffness (K)	Total Relative stiffness (ΣK)	Distribution Factors (K/ΣK)
B	BA	$\frac{I}{l} = \frac{2I}{4}$	$\frac{3I}{4}$	$2/3$
	BC	$\frac{3}{4}\frac{I}{l} = \frac{3}{4}\cdot\frac{I}{3}$		$1/3$

Moment Table

	A		B ($\frac{2}{3}$)	B ($\frac{1}{3}$)	C
	-45.00		+15.00	-13.33	+26.67
				-13.34	-26.67
	-45.00		+15.00	-26.67	0
			+7.78	+3.89	
	+3.89				
Final moments	-41.11		+22.78	-22.78	0



Free BM for AB & BC $= \frac{Wab}{L}$

Consider jt B; $M_{ba} = +15$; $M_{bc} = -26.67$

$M_{ba} + M_{bc} = +15 - 26.67 = -11.67$ KN·m

* For eq^{bm} @ jt B; condition is $M_{ba} + M_{bc} = 0$ ∴ Total correction of +11.67 KN·m should be applied @ B.

Dis factor @ B for BC = $1/3$

D·F @ B for BA = $2/3$

∴ Correction $C_{bc} = \frac{1}{3}(+11.67) = +3.89$ KN·m

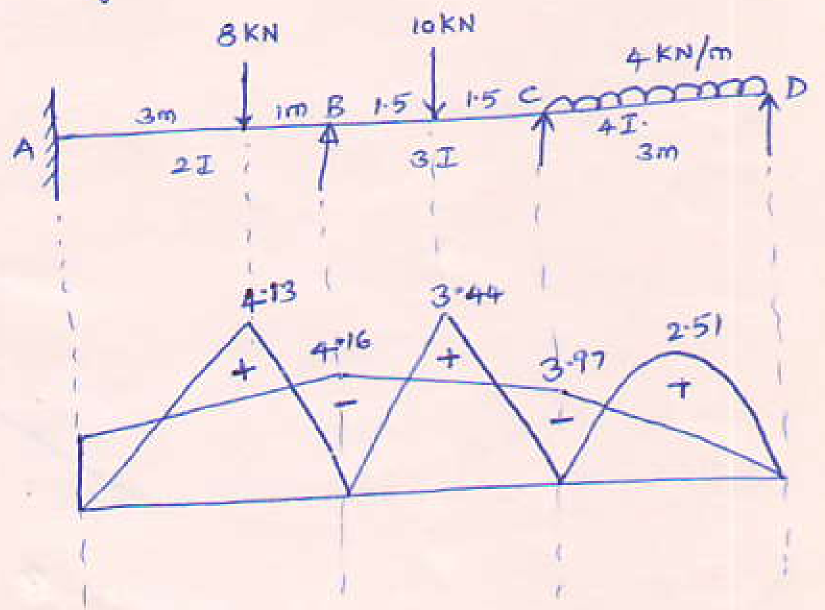
Correction $C_{ba} = \frac{2}{3}(+11.67) = +7.78$ KN·m

Carry Over Process

Moment of +7.78 KN·m has been applied @ B for span BA.

So moment of $\frac{+7.78}{2} = +3.89$ is carried over to A.

Prob: Determine support moments for continuous beam by M·D· method.



Sol^n: F·E·M

$\bar{M}_{ab} = -\frac{Wab^2}{l^2} = -1.5$ KN·m

$\bar{M}_{ba} = +\frac{Wba^2}{l^2} = +4.5$ KN·m

$\bar{M}_{bc} = -\frac{Wl}{8} = -3.75$ KN·m

$\bar{M}_{cb} = +\frac{Wl}{8} = +3.75$ KN·m

$\bar{M}_{cd} = -\frac{wl^2}{12} = -3.00$

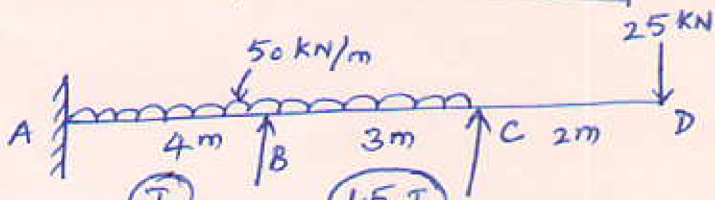
$\bar{M}_{dc} = +\frac{wl^2}{12} = +3.00$

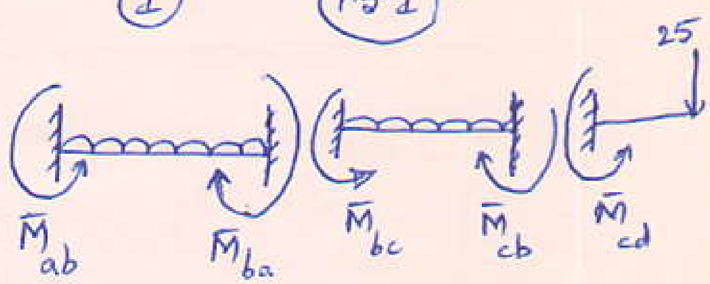
Distribution Factors

Jt	Member	Relative stiffness (K)	Total Rel. stiffness (ΣK)	D·F $K/\Sigma K$
B	BA	$2I/4 = I/2$	$3I/2$	$1/3$
	BC	$3I/3 = I$		$2/3$
C	CB	$3I/3 = I$	$2I$	$1/2$
	CD	$\frac{3}{4} \cdot \frac{4I}{3} = I$		$1/2$

A	B (1/3)	B (2/3)	C (1/2)	C (1/2)	D
-1.50	+4.50	-3.75	+3.75	-3.00	+3.00
				-1.50	-3.00
-1.50	+4.50	-3.75	+3.75	-4.50	0
	-0.25	-0.50	+0.38	+0.37	
-0.13		+0.19	-0.25		
	-0.06	-0.13	+0.13	+0.12	
-0.03		+0.06	-0.06		
	-0.02	-0.04	+0.03	+0.03	
-0.01		+0.02	-0.02		
	-0.01	-0.01	+0.01	+0.01	
-1.67	+4.16	-4.16	+3.97	-3.97	0

Prob: Find support moments by Moment distribution method.





Soln

$$\bar{M}_{ab} = \frac{-wl^2}{12} = -66.67 \text{ kN·m}$$

$$\bar{M}_{ba} = \frac{+wl^2}{12} = +66.67 \text{ kN·m}$$

$$\bar{M}_{bc} = \frac{-wl^2}{12} = -37.5 \text{ kN·m}$$

$$\bar{M}_{cb} = \frac{+wl^2}{12} = +37.5 \text{ kN·m}$$

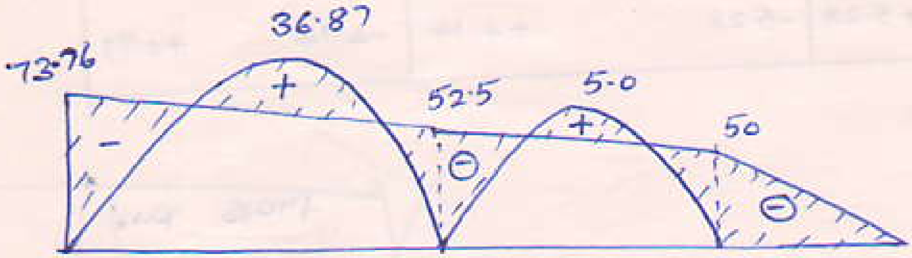
$$\bar{M}_{cd} = -25 \times 2 = -50 \text{ kN·m}$$

* Refer D-Factor table

A	B (2/5)	B (3/5)	C	C	D
		+25.00	+50.00	-50.00	
-66.67	+66.67	-37.50	+37.50		
		-18.75	-37.50		
-66.67	+66.67	-31.25	+50.00	-50.00	
	-14.17	-21.25			
-7.09					

Dis. factors

Jt	Member	Rel. stiffness (K)	Total relative stiffness (ΣK)	Distri factor K/ΣK
B	BA	$I/4 =$	$\frac{5I}{8}$	2/5
	BC	$\frac{3}{4} \cdot \frac{1.5I}{3}$		3/5



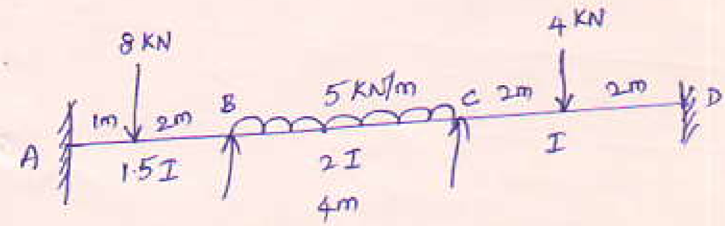
Prob:

$\bar{M}_{ab} = -\frac{Wab^2}{l^2} = -3.56$

$\bar{M}_{ba} = +1.78$

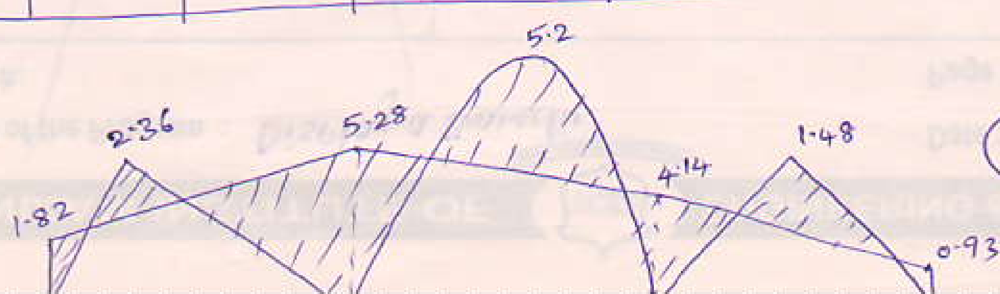
$\bar{M}_{bc} = -6.67$ $\bar{M}_{cb} = 6.67$

$\bar{M}_{cd} = -2.00$ $M_{dc} = +2.00$



Dis. factors

Jt	Mem	R.S (K)	T.R.S. (ΣK)	D.F K/ΣK
B	BA	$1.5I/3$	$\frac{2I}{2}$	1/2
	BC	$I/2$		1/2
C	CB	$\frac{1.5I}{3}$	$\frac{3I}{4}$	2/3
	CD	~~3/4~~ I/L		1/3



P.T.O

Prob :

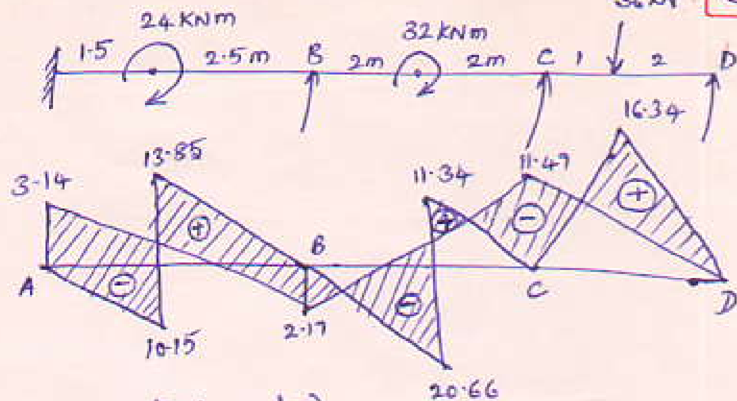
F.E.M

(Both are +ve)

$\bar{M}_{ab} = \frac{b(3a-l)}{l^2} M_o$ $a = 1.5$, $b = 2.5$, $M_o = 24$

$= +1.88$ kN·m

$\bar{M}_{ba} = \frac{a(3b-l)}{l^2} M_o = +7.88$ kN·m



IIIy $\bar{M}_{bc} = \frac{32}{4} = +8$ kNm ($\frac{M}{4}$ when moment is centre)

$\bar{M}_{cb} = \frac{32}{4} = +8$ kNm

$\bar{M}_{cd} = -\frac{Wab^2}{l^2} = -16$ kNm $\bar{M}_{dc} = +\frac{Wba^2}{l^2} = 8$ kNm

Dis. factors

Jt.	Mem	R.S	T.R.S	DF
B	BA	I/4	2I/4	1/2
	BC	I/4		1/2
C	CB	I/4	2I/4	1/2
	CD	3/4 I/L		1/2

AB	BA	BC	CB	CD	DC
	1/2	1/2	1/2	1/2	
+1.88	+7.88	+8.00	+8.00	-16.00	+8.00
				-4.00	-8.00
+1.88	+7.88	+8.00	+8.00	-20.00	0
	-7.94	-7.94	+6.00	+6.00	
-3.97		+3.00	-3.97		
	-1.50	-1.50	+1.98	+1.99	
-0.75		+0.99	-0.75		
	-0.49	-0.50	+0.38	+0.37	
-0.25		+0.19	-0.25		
	-0.09	-0.10	+0.12	+0.13	

6-A

	1/2	1/2	2/3	1/3	
-3.56	+1.78	-6.67	+6.67	-2.00	+2.00
	+2.44	+2.45	-3.11	-1.56	
+1.22		-1.56	+1.23		-0.78
	+0.78	+0.78	-0.82	-0.41	
+0.39		-0.41	+0.39		-0.20
	+0.2	+0.21	-0.26	-0.13	
+0.10		-0.13	+0.10		-0.07
	+0.06	+0.07	-0.07	-0.03	
-1.82	+5.28	-5.28	+4.14	-4.14	+0.93

Prob : Analyse the beam if support B & C sink by 2 mm & 7 mm

$E = 200\ KN/mm^2 \quad I = 2.5\times10^7\ mm^4$

F.E.M

Soln : $\bar{M}_{ab} = \bar{M}_{ba} = -\dfrac{6\times200\times2.5\times10^7\times2}{3^2\times10^9} = -6.67$

$\bar{M}_{bc} = \bar{M}_{cb} = \dfrac{-6\times200\times(2\times2.5)10^7(7-2)}{6^2\times10^9} = -8.33$

Joint	Member	R.S (K)	T.R.S (ΣK)	D.F
B	BA	$\frac{3}{4}\frac{I}{3}$ =	$\frac{7I}{12}$	3/7
	BC	$\frac{2I}{6}$ =		4/7
C	CB	$2I/6$	$\frac{25I}{48}$	16/25
	CD	$\frac{3}{4}\cdot\frac{I}{4}$		9/25

$\bar{M}_{cd} = \bar{M}_{dc} = +\dfrac{6\times200\times2.5\times10^7\times7}{4^2\times10^9} = +13.13\ KN\cdot m$

$\bar{M}_{ab} = -\left(\dfrac{wl^2}{12} + \dfrac{6EI\delta}{l^2}\right)$

$\bar{M}_{ba} = +\dfrac{wl^2}{12} - \dfrac{6EI\delta}{l^2}$

$\bar{M}_{bc} = \dfrac{-Wl}{8} + \dfrac{6EI\delta}{l^2}$

$\bar{M}_{cb} = +\dfrac{Wl}{8} + \dfrac{6EI\delta}{l^2}$

A	BA	BC	CB	CD	D
	3/7	4/7	16/25	9/25	
-6.67	-6.67	-8.33	-8.33	+13.33	+13.3
+6.67	+3.33			-6.56	-13.3
0	-3.34	-8.33	-8.33	+6.57	0
	+5.00	+6.67	+1.13	+0.63	
		+0.56	+3.33		
	-0.24	-0.32	-2.13	-1.20	
		-1.06	-0.16		
	+0.45	+0.61	+0.10	+0.06	
0	+1.89	-1.89	-5.96	+5.96	0