MSMF GATE CENTRE

Subject: Control Systems

Test 4: Root Locus Diagram (RLD) & Frequency response Analysis

(Solutions)

1. **Ans: (c)**
   Sol:
   RLD starts at a pole (k = 0) and terminates at a zero (k = ∞) of the OLTF G(S)H(S)

2. **Ans: (b)**
   Sol:
   k = ∞ RLD ends at open loop zeros, they are the closed loop poles for k = ∞.
   $\therefore$ j2, -j2, ∞ are the closed loop poles.

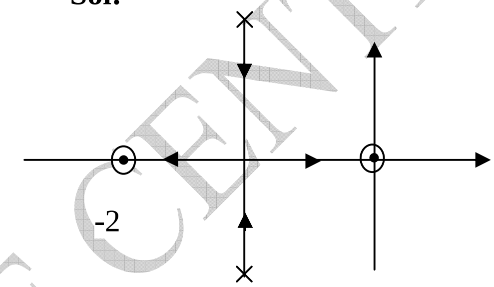
3. **Ans: (d)**
   Sol:
   Centroid always lies on the real axis
   Centroid (σ) =

$$\frac{\sum \text{real parts of poles of } G(s)H(s) - \sum \text{real parts of zero of} G(s)H(s)}{P - Z}$$

   which gives real quantity

4. **Ans: (d)**
   Sol:
   Break away point can lie any where in the s-plane
   E.g. G(s) H(s) = $\dfrac{K}{(s^2+2s+2)^2}$ Break away points are -2 $\pm$ j2

5. **Ans:(a)**
   Sol:



It is clear from the RLD at S = -2 arrival is 0^0.

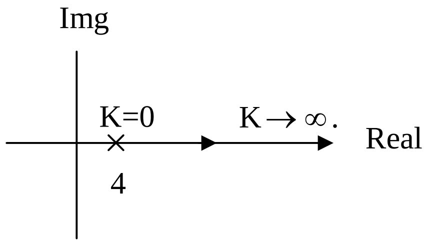
6. **Ans: (c)**
   Sol:
   OLTF = $\dfrac{K}{4-S} = \dfrac{-K}{(S-4)}$

   (or) Characteristics Equation

   $1 + \dfrac{K}{4-S} = 0$

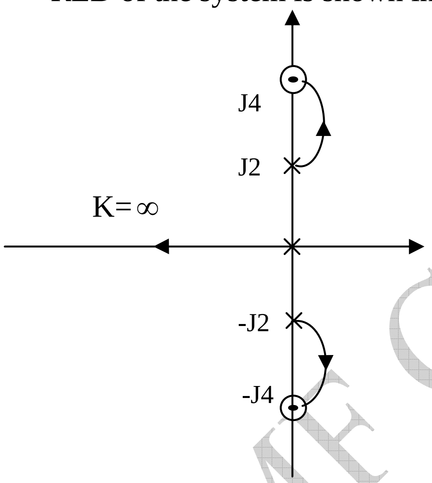
   4 – S + K = 0
   S = K + 4

   As K increases root moves towards right side of s-plane
   $\therefore$ RLD is



7. **Ans: (b)**
   Sol:
   It is clear from the RLD that the system is stable for only certain positive values of K
   from'0' the only answer is $0 < k < 10$

8. **Ans: (b)**
   Sol:
   RLD of the system is shown in fig below



It is clear from the RLD $S_1 = j3$ is not on the RLD, but $S_2 = -5$ is on the RLD

Method II

One may use the angle condition to verify the same.

9. **Ans: (a)** (correction in the question)
   Sol:

$$OLTF = \frac{CLTF}{1-CLTF} = \frac{\frac{k}{s+2+k}}{1-\frac{k}{s+2+k}} = \frac{k}{s+2}$$

$$K\Big|_{s=-6} = \frac{1}{\left(\frac{1}{S+2}\right)}$$

$$K = \left|S+2\right|_{S=-6}$$

$$K = \left|-6+2\right| = 4$$

K = 4

(Or) **Method 2**

Characteristic equation is s+2+k=0, s=-2 will give k=4

10. **Ans: (c)**
    Sol: It is clear from the given RLD that the option in (a) (b) (d) are not at all valid ∴ the only option is (c) which is correct.

11. **Ans: (d)**
    Sol:
    Characteristic equation is $1 + \frac{K(S+4)}{S(S^2+6S+13)}$

$S^3 + 6S^2 + 13S + KS + 4K = 0$

$S^3 + 6S^2 + (13 + K)S + 4K = 0$

S^3	1	13+K
S^2	6	4K
S^1	$\frac{6(13+K)-4K}{6}$	
S^0	4K	

S^1 row = 0, $\frac{6(13+K)-4K}{6} = 0$

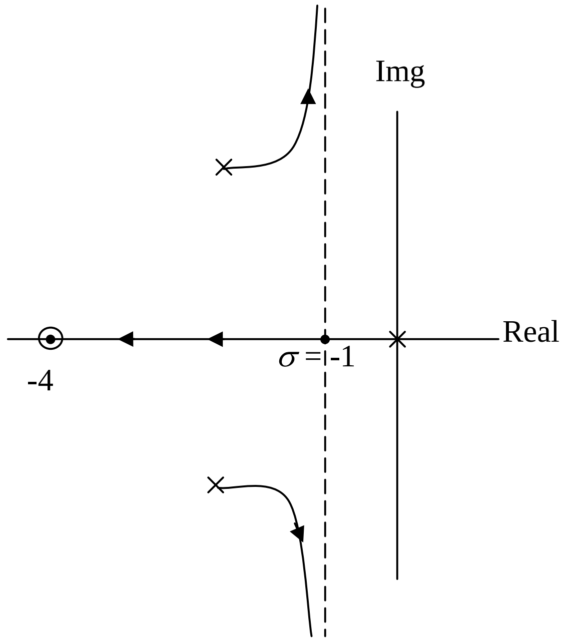
(13)(6) + 6K – 4K = 0

2K = (-13)(6)

K = -Ve

∴ For Positive values of 'K' $0 < k < \infty$, the RLD it will not intersect.

The RLD is given below



It is clear that it will not intersect the jω axis.

12. **Ans: (a)**

Sol:

1 + G(S)H(S) = S(S+2) + K(S +3) = 0

$$1 + \frac{K(S+3)}{S(S+2)} = 0$$

$$\therefore G(S)H(S) = \frac{K(S+3)}{S(S+2)}$$

$$\frac{d}{ds}\left[\frac{S+3}{S(S+2)}\right] = 0$$

$S^2 + 6s + 6 = 0$

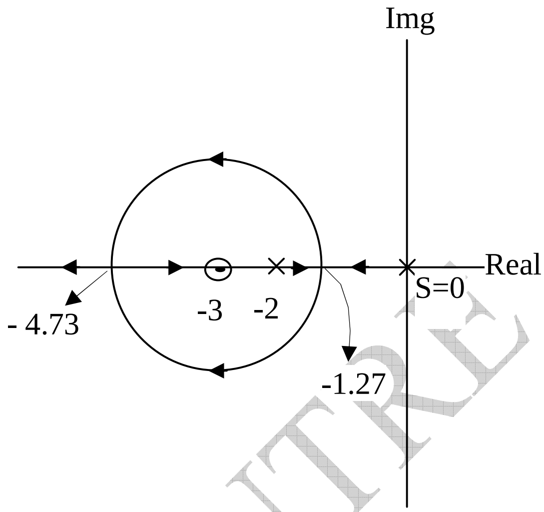
Roots are, $-3 \pm \sqrt{3}$

i.e. -1.27, -4.73, are the break away points.

13. **Ans: (b)**

Sol:

The RLD is given below



The diameter of the circle is the distance between the break away points

i.e (4.73 -1.27) = 3.46

14. **Ans: (b)**

Sol:

S^3	1	16
S^2	30	16K
S^1	$\frac{30(16)-16K}{30}$	
S^0	16K	

S^1 row = 0

30(16) – 16K = 0

K = 30

AE = $30S^2 + 16K = 0$

$30S^2 + 16(30) = 0$

$S = \pm j4$

$\therefore$ RLD intersection at $S = \pm j4$

15. **Ans: (b)**

Sol:

1 + G(S)H(S) = 0

$S^3 + 30S^2 + 16S + 16K = 0$

$$1 + \frac{16K}{S^3 + 30S^2 + 16S} = 0$$

$$G(S)H(S) = \frac{16K}{S(S^2 + 30S + 16)}$$

P = 3, Z = 0

∴ No of asymptotes = |P-Z|

=|3-0| = 3

16. **Ans: (d)**

Sol:

Frequency response is the steady state output of at system to the sinusoidal input.

17. **Ans: (c)**

Sol:

Eg: Transportation lag = e^{-ST_D}

It introduces the negative phase, hence Phase and Gain margins of the system decreases

∴ Stability decreases.

18. **Ans: (b)**

Sol:

GM = $\dfrac{1}{|G(j\omega_{pc})H(j\omega_{pc})|}$

It is clear from the expression the gain is decreased then GM increases,, hence if the gain is made half, GM increase by '2'.

19. **Ans:(a)**

Sol:

f = 0.1591 Hz

$\omega = 2\pi f = 2\pi(0.1591) = 1$rad/sec

$$\left|\frac{1}{(j\omega)^2 + j\omega + 1}\right| = \left.\frac{1}{\sqrt{(1-\omega^2)^2 + \omega^2}}\right|_{\omega=1} = 1$$

(or) 20log1 = 0 dB

20. **Ans: (b)**

Sol:

If peak overshoot is zero, implies that damping ratio, $\xi \geq 1$

Resonant peak for $\xi \geq \dfrac{1}{\sqrt{2}}$ is '1'

21. **Ans: (b)**

Sol:

Stable range of k is 0 < k < 20

GM = $\dfrac{\text{The value of k for marginal stability}}{\text{Required value of K}}$

GM = $\dfrac{20}{10}$ = 2

(or) GM in dB = 20log2 = 6dB

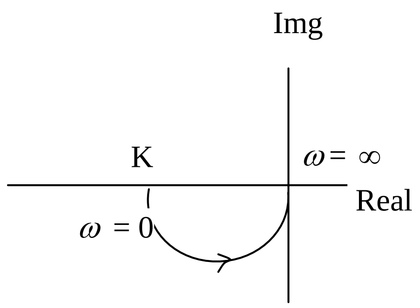
22. **Ans: (c)**

Sol:

Eg. OLTF = $\dfrac{K}{S-2}$

$= \dfrac{K}{j\omega - 2} = \dfrac{K}{\sqrt{\omega^2 + 4}}$ - (180-tan$^{-1}\dfrac{\omega}{2}$)

The Polar plot is



23. **Ans:(c)**

Sol:

Eg let us consider a system with G(S)H(S) = $\dfrac{K}{S(S+2)}$

This system is stable for 0 < K < ∞

24. **Ans: (d)**

Sol:

System is marginally stable

∴ GM = 1 (or) 0 dB

& PM = 0^0

25. **Ans: (a)**
Sol:
It is a polar plot of type '0' system
∴ Option "**a**" is correct

26. **Ans: (c)**
Sol:
G(S)H(S) has a pole at the origin and poles on the jω axis
∴ option '**c**' is correct

27. **Ans: (d)**
Sol:
Corner frequencies are $= \log\omega_1 = 0$

$\omega_1 = 1$

$\therefore T_1 = \frac{1}{1}$

and $\log\omega_2 = 1$

$\omega_2 = 10$

$\therefore T_2 = \frac{1}{10}$

The starting point frequency $\log\omega = -1$

$\omega = 0.1$

Initial slope is 20 dB/dec hence portion of the TF = G(S)H(S) = Ks
At $\omega_1 = 1$ slope changes to '0' dB. i.e. -20 dB/dec is added to initial slope +20 dB/dec, implies there is a presence of real pole
Hence portion of the TF G(S)H(S) = $\frac{KS}{1+ST_1}$

At $\omega_2 = 10$ slope become – 20dB/dec implies another pole is added

$$\therefore \text{Total TF } G(S)H(S) = \frac{Ks}{(1+sT_1)(1+sT_2)}$$

$|K| + |S| = 6\text{dB}/\ \omega = 0.1$

$(20\log K + 20\log\omega)_{\omega=0.1} = 6\text{dB}$

K = 20

$$\therefore \text{ TF } G(S)H(S) = \frac{20S}{(1+S)(1+S.\frac{1}{10})}$$

$$= \frac{20S}{(1+S)(1+0.1S)}$$

28. **Ans: (b)**
Sol:

$$\text{CLTF} = \frac{G(S)}{1+G(S)(H(S)}$$

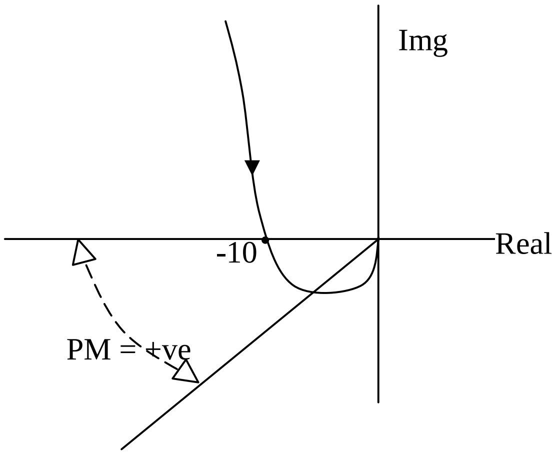
$$\text{CLTF} = \frac{20S}{(1+S)(1+0.1S)+20S}$$

$|\text{CLTF}|_{\omega=0} = 0$

29. **Ans: (d)**
Sol:
For a > 1, (-1, j0) is enclosed once in the ccw direction
∴ N = 1, (from the plot)
& P = 1 (Given)

N = P – Z
Z = P – N
Z = 1 -1 = 0
Z = 0 System is stable
a > 1, hence a = 1.5

30. **Ans: (c)**
Sol:
For a = 10
PM is +Ve
∴ PM = 45^0


Img
-10
Real
PM = +ve